

Golden

Notes

Math &

Physics

Akshansh Chaudhary

Parabola $e=1$

$$y^2 = 4ax$$

Tangent at $(\frac{a}{m^2}, \frac{2a}{m})$
 $y = mx + a/m$ Ever tangent
Tangent at $(at^2, 2at)$
 $ty = x + at^2$

Normal at $(at^2, 2at)$
 $tx + y = 2at + at^3$

Chord joining t_1 & t_2

$y(t_1 + t_2) = 2x + 2at_1 t_2$
It passes through focus
iff $t_1 t_2 = -1$

For a line $lx + my + n = 0$
to be a tangent
 $ln = m^2 a$

For $y = mx + c$ to be
tangent, $c = a/m$
 t_1 & t_2 intersect at.
[$2a(GM)$, $2a(AM)$]

$$\therefore c \in [2a(\frac{t_1 + t_2}{2}), 2a(\frac{t_1 t_2}{2})]$$

General notations:

$$S = ax^2 + by^2 + 2gx + 2fy + c$$

$$S_1 = ax_1^2 + by_1^2 + 2gx_1 + 2fy_1 + c$$

$$S_{11} = ax_1^2 + by_1^2 + 2gx_1 + 2fy_1 + c$$

$$S_{12} = ax_1 x_2 + by_1 y_2 + g(x_1 + x_2) + f(y_1 + y_2) + c$$

If $S=0$ is a conic & $A(x_1, y_1)$ & $B(x_2, y_2)$
are 2 pts. on it. If $S_1=0$ & $S_2=0$ are not
||, then eqn of chord $\overline{AB} = S_1 + S_2 = S_{12}$

Combined eqn of pair of tangents drawn
from an ext. pt. (x_1, y_1) to conic $S=0$
is $S_1^2 = SS_{11}$

In any conic, two tangents or 2 lines
are \perp to each other iff coeff. of $x^2 +$
coeff. of $y^2 = 0$

Angle b/w $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$
or $ax^2 + 2hxy + by^2 = 0$ is given as

Hyperbola ($e > 1$)

$$b^2 = a^2(e^2 - 1)$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Vertices $(\pm a, 0)$

Directrix $x = \pm \frac{a}{e}$

Length of LR = $\frac{2b^2}{a}$

Coordinates of LR = $(ae, \pm \frac{b^2}{a})$

Tangent at $(a \sec \theta, b \tan \theta)$

$$= \frac{x}{a} \sec \theta - \frac{y}{b} \tan \theta = 1$$

Normal at $\theta =$
 $a x \sec \theta + b y \cot \theta = a^2 + b^2$

Ever tangent

$$y = m x \pm \sqrt{a^2 m^2 - b^2}$$

line $lx + my + n$ is tangent

$$\text{iff } l^2 a^2 - m^2 b^2 = n^2$$

line $y = mx + c$ is tangent

$$\text{iff } y - mx - c^2 = a^2 m^2 - b^2$$

Chord joining θ_1 & θ_2 =

$$\frac{x}{a} \cos \left(\frac{\theta_1 - \theta_2}{2} \right) - \frac{y}{b} \sin \left(\frac{\theta_1 + \theta_2}{2} \right) = \cos \left(\frac{\theta_1 + \theta_2}{2} \right)$$

If $S=0$ is eqn of hyperbola

& (x_1, y_1) an external pt.,

then, $S_{11} < 0$.

Director circle eqn $x^2 + y^2 = a^2 - b^2$

• For $S = ax^2 + by^2 + 2hx + 2fy + c = 0$,

$$\Delta = h^2 - ab, \Theta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$$

Now,

✓ For S to represent intersecting lines,
 $\Delta > 0$ & $\Theta = 0$

✓ For S to represent || lines,
 $\Delta = 0, \Theta = 0$ & $bg^2 = af^2$

• Pts. from which 2 tangents to a
given conic $S=0$ are \perp lie on

(i) directrix if $S=0$ is parabola

(ii) circle (director circle) if $S=0$ is
ellipse or hyperbola with transverse
axis $>$ conjugate axis.

• Eqn of chord of contact for any
conic: $S_1 = 0$

$$\Theta = \tan^{-1} \left(2 \sqrt{\frac{h^2 - ab}{a + b}} \right)$$

MATHS

• N arithmetic means b/w x & y ,
 $c.d.(d) = \left(\frac{y-x}{N+1}\right)$

• N geometric means b/w x & y ,
 Common ratio (r) = $\left(\frac{y}{x}\right)^{1/(n+1)}$

• N harmonic means b/w x & y ,
 Common diff. (d) = $\frac{x-y}{\frac{xy}{2}(n+1)}$

• Sum of ∞ terms of an AGP :-
 $S_{\infty} = \frac{ab}{1-r} + \frac{drb}{(1-r)^2}$

• If a is sum of N terms of
 AGP :-

$$S_n = \frac{ab}{1-r} + \frac{drb(1-r^{n-1})}{(1-r)^2} - \frac{[a+(n-1)d]b r^n}{1-r}$$

$$2 \sum_{i < j} a_i a_j = (a_1 + a_2 + \dots + a_n)^2 - (a_1^2 + a_2^2 + \dots + a_n^2)$$

• If α & β are roots of a
 quad. eqn $a x^2 + bx + c = 0$,
 $\alpha - \beta = \frac{\sqrt{D}}{a}$

• Coeff. of x^n in $(1-x)^{-n}$
 $= n+r^{-1} C_r$

• No. of dearrangements

$$n! \left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots + (-1)^n \frac{1}{n!} \right]$$

• Greatest term in expansion
 $(a+x)^n := \alpha : \frac{n+1}{1+|\frac{a}{x}|}; \alpha \in \mathbb{Z}$

& $\xi = \alpha - 1 \pm \alpha$ if $\alpha \in \mathbb{Z}$.

• If $(\sqrt{A} + B)^n = I + f$
 $n = \text{odd}$, then, $(I+f) \cdot f = K^n$

where $A - B^2 = K > 0$ & $\sqrt{A} - B < 1$.
 $n = \text{even}$, $(I+f)(1-f) = K^n$

• $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \dots$; $e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$

• $a^x = 1 + \frac{x \ln a}{1!} + \frac{x^2 (\ln a)^2}{2!} + \dots$

• $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots - \frac{x^4}{4} + \dots$

• Periodic matrix :- $A^{K+1} = A$; K: Period

• Ht. of capillary tube rise (h) = $\frac{2 S \cos \theta}{\rho g r}$

• For a plane progressive wave, amplitude (A) doesn't change.

$$F_{\text{viscous}} = 6\pi \eta \delta V$$

$$F_{\text{viscous}} = \eta A \frac{dv}{dx}$$

• Poisson's Ratio : - $-\frac{\Delta d/d}{\Delta l/l} = \frac{\text{Transverse strain}}{\text{Longitudinal strain}}$

• Thermal stress (σ) = $Y \alpha (\Delta \theta)$

$$\text{Speed of effluo} (v) = \sqrt{\frac{2gh}{1 - \frac{a^2}{A^2}}}$$

• Both closed / Both open $\Rightarrow \text{dof} = n \left(\frac{V}{2l}\right)$
 $n \in \mathbb{Z}^+$

• One closed, one open, $\Rightarrow \text{dof} = n \left(\frac{V}{4l}\right); n = \text{odd}$

• Degree of freedom (f) = $3N - R$
 $N: \text{no. of particles}; R: \text{no. of bonds (relations)}$

$$C_v = \frac{f}{2R}$$

$$P_{\text{avg}} = 2\pi^2 J^2 A^2 v \mu; I = \frac{P}{A} = \frac{E}{At}; A: \text{Area}$$

• L: Level of soundness : $10 \log \frac{I}{I_0}$; $I_0 = 10^{-12} \text{ Wb m}^{-2}$

• Particle vel: $A(\omega)$, Wave vel: λ

$$\mu = \frac{m}{T} = \beta A$$

• Energy (say $\frac{1}{2} mv^2$); $v = A\omega \Rightarrow E = \frac{1}{2} m A^2 \omega^2$
 $\Rightarrow E \propto (\text{Amplitude})^2 (\text{Frequency})^2$

• For sonometer, $n \propto \frac{1}{\sqrt{m}}$; m: mass
 $n: \text{no. of antinodes}$

$$V_{\text{particle}} = -V_{\text{wave}} \frac{\partial Y}{\partial t}$$

• Beat frequency ($\bar{f}_2 - \bar{f}_1$): No. of times max. intensity occurs in 1 sec. or no. of maxima in 1 sec.

• For an expression to describe wave motion,

$$\frac{\partial^2 Y}{\partial x^2} = (\text{constant}) \frac{\partial^2 Y}{\partial t^2}$$

• Distance b/w 2 consecutive nodes or antinodes is $\frac{\lambda}{2}$ and distance b/w a node and an antinode is $\lambda/4$.

• For sonometer, $\bar{f} = \frac{nV}{2l}$

$$* F = 32 + \frac{9}{5}c$$

Multiply a complex no. by i
rotates it anti clockwise by $\frac{\pi}{2}$ i.e.,
argument increases by $\frac{\pi}{2}$.

During compression, \uparrow in P is
more in adiabatic than isothermal.

During expansion, the \downarrow in P is
more in adiabatic than isothermal.

Hydration energy & charge
Lattice energy $\propto \frac{1}{\text{size}}$

Among isoelectronic species,
ionic size $\propto \frac{1}{\text{At. no.}}$

cation < neutral < anion.

Hyperconjugation involves overlap of
 $\sigma - \pi$ orbitals.

If algebraic sum (\perp distance)
of n points to the fixed line
is zero, then, line passes through
a fixed pt. having coordinates
(AM_x, AM_y): AM: Arithmetic Mean.

For $a_1x+b_1y+c_1=0$ & $a_2x+b_2y+c_2=0$
cuts coordinate axis at concyclic
points, then,

$$a_1a_2=b_1b_2; \text{ or } a_1b_2+a_2b_1=0$$

Projection of \vec{b} on \vec{a}

$$\text{Scalar: } |\vec{b}| \vec{a} \cdot (\vec{a} \cdot \vec{b}) \text{ or } |\vec{b}| |\vec{a}| \cos\theta$$

Projection of normal to \vec{b} (given vector)

$$\text{Scalar: } |\vec{a} - (\vec{a} \cdot \vec{b}) \vec{b}|$$

$$\text{Vector: } \vec{a} - |\vec{a}| \cdot \vec{b} \cos\theta$$

Volume of tetrahedron = $\frac{1}{6} \text{STP}$

Vol. of trigonal triangular prism = $\frac{1}{2} \text{STP}$

Reciprocal sys of vectors:-

$$(\vec{a})^{-1} = \frac{\vec{b} \times \vec{c}}{\text{STP}}$$

$$n+1 C_{z+1} = \left(\frac{n+1}{z+1} \right)^n C_z = \left(\frac{n+1}{z+1} \right)^{n-1} C_{z-1}, \dots$$

$$\bullet \frac{C_n}{C_{n-1}} = \frac{n-z+1}{z}; \frac{C_{z+1}}{C_z} = \frac{n-z}{z+1}; C_z \cdot C_s = C_s \cdot C_z$$

To find sum of binomial coeff. of the
expansion $(1+x)^n$ with 'k' no. of missing
terms; substitute $(k+1)$ roots of unity in
place of x on both sides of equation.
Then, add the $(k+1)$ terms obtained.
The result is answer.

$$\bullet \frac{N}{M} = \frac{\text{Normality}}{\text{Molarity}} = \frac{W}{EW} = n\text{-factor}$$

$$\bullet PV = \frac{1}{3} m N_A (\bar{c})^2 = ; \bar{c} : \text{Moles}$$

$$\bullet E = \frac{3}{2} K_b T; K_b = \frac{R}{N_A}$$

$$\bullet Z = \frac{V_m}{V_{\text{ideal}}} = \frac{PV_m}{RT} = \frac{P_{\text{real}}}{P_{\text{ideal}}}$$

$$\bullet (P + \frac{n^2 a}{V^2})(V - nb) = nRT$$

$$\bullet Z = \frac{1}{V_m} \left(b - \frac{a}{RT} \right) + 1 : \text{Solving Virial eq^n under approxim^n}$$

$$\bullet T_B = \frac{a}{Rb}; T_B: \text{Boyle Temp.}$$

$$\bullet T_{\text{critical}} = \frac{8a}{27Rb}; P_{\text{critical}} = \frac{a}{27b^2}; V_{\text{critical}} = 3b$$

$$\bullet \text{No. of spectral lines} = \frac{n(n-1)}{2}$$

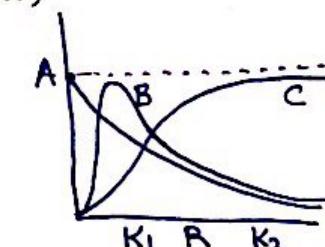
$$\bullet mvr = \sqrt{l(l+1)} h = \left(\sqrt{l(l+1)} \right) \frac{h}{2\pi}$$

$$\bullet \text{For Parallel Rxn} \quad A \xrightarrow{k_1} B \xrightarrow{k_2} C \quad \frac{[B]}{[C]} = \frac{k_1}{k_2}$$

For Consecutive reaction,

$$A \xrightarrow{k_1} B \xrightarrow{k_2} C \quad t_{\max} = \frac{1}{k_1+k_2} \ln \left(\frac{k_1}{k_2} \right)$$

$$[B]_{\max} = A_0 \left(\frac{k_2}{k_1} \right)^{\frac{k_1}{k_1+k_2}}$$



For Opposing reactions,

$$A \xrightarrow{k_1} Z \xleftarrow{k_2} (A) \quad ; \quad K = \frac{k_1}{k_2 \alpha k_{-1}}$$

$$\frac{dx}{dt} = \text{Rate of form}^n \text{ of } Z = k_1(A_0 - x) - k_{-1}(x)$$

$$\bullet K_b = \frac{M_f R T_b^\circ}{\Delta H_{1\text{mv}}^\circ}; K_f = \frac{M_f R T_f^\circ}{\Delta H_{1\text{inf}}^\circ}$$

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- Sum of ∞ terms of an AGP:-

$$S_{\infty} = \frac{ab}{1-r} + \frac{dbr}{(1-r)^2}$$

- If a & b sum of N terms of AGP:-

$$S_n = \frac{ab}{1-r} + \frac{dbr(1-r^{n-1})}{(1-r)^2} - \frac{[a+(n-1)d]br^n}{1-r}$$

$$2 \sum_{i < j} a_i a_j = (a_1 + a_2 + \dots + a_n)^2 - (a_1^2 + a_2^2 + \dots + a_n^2)$$

- If α & β are roots of a quad. eqn $ax^2 + bx + c = 0$,

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- No. of dearsay

$$n! \left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots (-1)^n \frac{1}{n!} \right]$$

- Greatest term in expansion

$$(a+x)^n := \alpha : \frac{n+1}{1+|\frac{a}{x}|}; \alpha = [a];$$

& $\beta = \alpha - 1 \& \alpha \text{ if } \alpha \in \mathbb{Z}$.

- If $(\sqrt{A} + B)^n = I + f$

$$n = \text{odd}, \text{then, } (I+f) \cdot f = K^n$$

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- Periodic matrix:- $A^{K+1} = A$; K : Period

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- Poisson's Ratio:- $-\Delta d/d // \Delta l/l$

- Thermal stress (σ) = $Y \propto (\Delta \theta)$

$$\text{Speed of efflux}(v) = \sqrt{\frac{2gh}{1 - \frac{a^2}{A^2}}}$$

$$\text{Both closed/Both open} \rightarrow \text{DoF} = n \left(\frac{v}{2l} \right) \\ n \in \mathbb{Z}^+$$

$$\text{One closed, one open} \rightarrow \text{DoF} = n \left(\frac{v}{4l} \right); n = \text{odd.}$$

$$\text{Degree of freedom (f)} = 3N - R \\ N: \text{no. of particles}; R: \text{no. of bonds (relations)}$$

$$C_v = \frac{f}{2R}$$

- 1 AU - Astronomical unit = Distance b/w centre of earth and centre of Sun
 $= 1.496 \times 10^{11} \text{ m} \approx 1.5 \times 10^{11} \text{ m}$
- 1 Par Sec = 1 Parallactic second = It is distance from an arc of 1 AU that subtends 1" angle.
 1 Par Sec = $3 \times 10^{18} \text{ m}$.
- Angular fringe separation (θ) = $\frac{\lambda}{d}$
- Variation of g :-
 On surface = $g_0 = \frac{G_1 M_e}{R_e^2}$
 At a ht. (h) = $g_0 \frac{R_e^2}{(R_e+h)^2} \approx g_0 \left(1 - \frac{2h}{R_e}\right)$
- At a depth (d) = $g_0 \left(1 - \frac{d}{R_e}\right)$
 Due to rot. $n := g_0 - (\omega^2 \cos^2 \theta) R_e$
- Power of radiation received by a surface at temp. (T_0) by any other surface at temp. ($T > T_0$) and distance d is given as $P \propto \frac{T^4}{d^2}$
- Acc. to Rayleigh's scattering law,
 Intensity of light scattered (I) $\propto \frac{1}{\lambda^4}$
- If ($N+1$) polaroids are placed, so that their axes are equally spaced, then intensity of light after passing through all polaroids :-
 $I = I_0 \left(\cos \frac{\pi}{2N}\right)^{2N}$
- If a vehicle is overtaking a car (fitted with convex mirror - driving mirror) of focal length f , then speed of image of vehicle (V_i) formed by the mirror (as seen by person in the car) is :-
 $V_i = -\frac{f^2}{(v-f)^2} \times (\text{rel. speed of vehicle wrt car})$
 v : distance of vehicle from car.
- Acidic character $\propto E_N$ of central atom
- For bond to be covalent:- Lanthanides period
 Large size of anion
 Small size of cation
 highly charged cation or anion or both.
- When particles are falling in a viscous fluid under gravity, they attain a terminal velocity (V_s) s.t. their f_s combined with buoyancy balances wt. (gravitational force).
 Then, $V_s = \frac{2}{9} \frac{(S_{\text{particle}} - S_{\text{fluid}}) g R^2}{\eta} \quad \text{viscosity}$
 $(V_s \delta g = 6\pi \eta R V_s)$
- The terminal velocity is attained vertically upwards : $S_{\text{particle}} < S_{\text{fluid}}$
 vertically downwards : $S_{\text{particle}} > S_{\text{fluid}}$.
- Electron (e^- , p^-) Proton α -particle ($\alpha, \alpha^{2+}, He^{2+}$)
- q: $-1e \quad +1e \quad 2e$
 $-1.6 \times 10^{-19} C \quad +1.6 \times 10^{-19} C$
 $-4.8 \times 10^{-10} \text{ esu}$
- m: $5.48 \times 10^{-4} u \quad 1u \quad 4u$
 $9.1 \times 10^{-31} \text{ Kg} \quad 1.6 \times 10^{-27} \text{ Kg}$
- Lateral shift in a glass prism :-
 $d = \frac{t}{\cos r} \sin(i-r)$; t : thickness of slab
- Units of activity of radioactive sample :-
 SI = Becquerel (Bq) or disintegrations per sec (dps)
 $1 \text{ Curie (Ci)} = 3.7 \times 10^{10} Bq = 3.7 \times 10^{10} \text{ dps}$
 $1 \text{ Rutherford (Rd)} = 10^6 Bq = 10^6 \text{ dps}$
- Wein's Displacement law
 $\lambda_{\max} T = b = 2.89 \times 10^{-3} \text{ mK}$
- Newton's Law of Cooling
 $\frac{dQ}{dt} + ms \frac{dT}{dt} = -\frac{\Delta T}{t/KA} = -\frac{\Delta T}{t/KA} = e^{-A(4T_0^3)t/KA} = hA(-\Delta T) = \frac{KA}{t}(-\Delta T)$
- Time taken for cooling
 $t = \frac{k}{KA} \ln \frac{\Theta_1 - \Theta_0}{\Theta_2 - \Theta_0}; \Theta = \text{Temp.}$
- TP of simple pendulum of length comparable to radius of earth
 $T = 2\pi \sqrt{\frac{1}{g(t+\frac{1}{R})}}$
- For n^{th} order axn, a
 $k_t = \frac{1}{n-1} \left[\frac{1}{[CA]^{n-1}} - \frac{1}{[A_0]^{n-1}} \right]; k = \left(\frac{1}{[\text{conc}]} \right)^{n-1} \left(\frac{1}{t} \right)$
- $K_p = K_c (RT)^{\Delta n}$
- E_N of central atom \propto Bond angle of compd.
- Acidic character \propto size : In group.
- \sum of all the n digit nos. that can be formed from n distinct digits (non zero)
 $= (\text{Sum of all digits})(n^{n-1})(1 + 10 + 10^2 + \dots + 10^{n-1})$

* For ungrouped data

$$\text{Mean} = \frac{\sum x_i}{n}; \text{Mean deviation} = \frac{\sum |x_i - \bar{x}|}{n}$$

$$\text{Median} := M, \text{say}, \text{ Mean deviation} = \frac{\sum |x_i - M|}{n}$$

* For discrete frequency distribution

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i}, \text{ Mean deviation} = \frac{\sum f_i |(x_i - \bar{x})|}{\sum f_i \text{ i.e total frequency (N)}}$$

$$\text{Median} = M, \text{say}, \text{ Mean deviation} = \frac{\sum f_i (|x_i - M|)}{\sum f_i}$$

* For continuous frequency distribution

x_i = Mid pt. of interval given.. Rest is same.

* Variance :- Replace $|x_i - \bar{x}|$ or $|x_i - M|$ by (σ^2) $(x_i - \bar{x})^2$ or $(x_i - M)^2$ in every formula given alone.

* Standard deviation = $\sigma = \sqrt{\text{Variance}}$

* Another formula for std. deviation =

$$\text{ungrouped: } \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n} \right)^2$$

$$\text{discrete: } \frac{\sum f_i (x_i)^2}{n} - \left(\frac{\sum f_i x_i}{n} \right)^2$$

* Coeff. of variation = $\frac{\sigma}{\bar{x}} \times 100 = C.V. \quad [\frac{\sigma}{\bar{x}}: \text{std. deviation}] \quad [\bar{x}: \text{Mean}]$